IDENTIFYING PRISMS AND PYRAMIDS

A ________ is a solid with two parallel congruent bases joined by faces that are parallelograms. A prism is named by the shape of its ________.

CHECK FOR UNDERSTANDING

Name each prism. In each prism, identify a base, a face, an edge, and a vertex.

1

2

3
A ___________ is a solid with a base that is a polygon and sides that are ___________ and meet at the top. A pyramid is named by the shape of its ___________.

**CHECK FOR UNDERSTANDING**

Name each pyramid.

- Triangular Pyramid
  - Triangular Base
  - 3 Triangular faces

- Rectangular Pyramid
  - Rectangular Base
  - 4 Triangular faces

- Hexagonal Pyramid
  - Hexagonal base
  - 6 triangular faces
Finding the areas of rectangles, triangles, and trapezoids

**Formulas**

**Rectangle**
- Area of rectangle
  \[ A = \ell \cdot w \]  

**Triangle**
- Area of triangle
  \[ A = \frac{1}{2} \cdot b \cdot h \]

**Trapezoid**
- Area of trapezoid
  \[ A = \frac{1}{2} \cdot h \cdot (b_1 + b_2) \]

**Example 1**: Find the area.

1.  
   \[ \text{Rectangle} \]
   \[ \ell = 9 \text{ cm}, \quad w = 4 \text{ cm} \]

2.  
   \[ \text{Triangle} \]
   \[ b = 12 \text{ m}, \quad h = 10 \text{ m} \]

3.  
   \[ \text{Triangle} \]
   \[ b = 7 \text{ ft}, \quad h = 10 \text{ ft} \]

4.  
   \[ \text{Trapezoid} \]
   \[ b_1 = 6 \text{ in.}, \quad b_2 = 6.5 \text{ in.}, \quad h = 10 \text{ in.} \]
Finding the Volumes of Rectangular Prisms

**Example 2**: Find the volume of each solid.

1. 
   - **Cube**: Volume = edge \( \times \) edge \( \times \) edge
   - **Rectangular Prism**: Volume = length \( \times \) width \( \times \) height

2. 
   - **Cube**: Volume = edge \( \times \) edge \( \times \) edge
   - **Rectangular Prism**: Volume = length \( \times \) width \( \times \) height

**Guided Practice**

a.) 
   - **Cube**: Volume = edge \( \times \) edge \( \times \) edge

b.) 
   - **Cube**: Volume = edge \( \times \) edge \( \times \) edge
NETS OF SOLIDS

A net is a plane figure that can be folded to make a _______________.

<table>
<thead>
<tr>
<th>Solid</th>
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<td>Cube</td>
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<tr>
<td>Rectangular Prism</td>
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<td>Square Pyramid</td>
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<td><img src="image" alt="Triangular Pyramid Diagram" /></td>
<td><img src="image" alt="Triangular Pyramid Net" /></td>
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FINDING SURFACE AREA OF CUBES AND RECTANGULAR PRISMS

The surface area of a solid is the area of its ________, or the ________ of the area of every face of the solid.

**EXAMPLE 3**: A wooden cube has edges measuring 5 centimeters each. Find the surface area of the cube.

The surface area of a cube is the area of the net of the cube. The net shows the 6 square faces of the cube.

If \( S \) is the surface area, and \( e \) is the length of an edge, then \( S=6e^2 \).
EXAMPLE 4: Find the surface area of a rectangular prism that is 12 inches long, 8 inches wide, and 4 inches high.

To find the surface area, draw a net of the rectangular prism. The surface area of the prism is the area of its net. The opposite faces of a rectangular prism are congruent. In the net below, the congruent faces are the same color.

The surface area of a rectangular prism can be found using the formula:

\[
\text{Surface area} = 2lw + 2lh + 2wh
\]
a) A cube has edges measuring 6 centimeters each. Find the surface area of the cube.

b) A rectangular prism measures 7 inches by 5 inches by 10 inches. Find the surface area of the prism.
FINDING SURFACE AREA OF TRIANGULAR PRISMS AND PYRAMIDS

The surface area of a triangular prism is the area of its net. A triangular prism has 3 _____________ faces and 2 congruent _____________ bases.

EXAMPLE 5: The triangular prism shown has three rectangular faces. Its bases are congruent isosceles triangles. Find the surface area of the triangular prism.

![Diagram of a triangular prism with dimensions labeled: 5 cm, 6 cm, 11 cm, 4 cm.]

![Diagram of a net of a triangular prism with dimensions labeled: 5 cm, 5 cm, 6 cm, 11 cm, 4 cm.]
EXAMPLE 6: This pyramid has a square base measuring 10 inches on each side. It has four faces that are congruent isosceles triangles. The height of each triangle is 12 inches. Find the surface area of the pyramid.
GUIDED PRACTICE

a) The triangular prism shown has three rectangular faces. Its bases are congruent right triangles. Find the surface area of the triangular prism.

b) Alicia makes a pyramid that has an equilateral triangle as its base. The other three faces are congruent isosceles triangles. She measures the lengths shown on the net of her pyramid. Find the surface area.
RECOGNIZING CYLINDERS, CONES, SPHERES AND PYRAMIDS

A solid cylinder has a curved surface and two ___________ bases that are ___________ circles. The radius of a cylinder is the radius of one of its bases. The ________ of a cylinder is the perpendicular distance between two parallel _________.

A cylinder has some things in common with a prism. Both solids have two bases. But the bases of a cylinder are circle instead of a polygon, and a cylinder has a curved surface instead of flat faces.

The cylinder has the net shown below. The net of a cylinder is made up of two circles for the two bases and a rectangle for the curved surface.
A ________ has a circular base, a cured surface, and one ______________. The curved surface of the cone is also called the lateral surface.

The height of a cone is the distance from the ____________ to the center of the base. The __________height is the distance from the vertex to any point on the circumference of the base.

The net of a cone is made up of a circle for the base and a portion of a circle for the lateral surface. The cone has the net shown below.
A ________________ has a curved surface. Every point on the surface is an equal
distance from the center of the sphere. The distance from the center of a sphere
to any point on its surface is called the ______________ of the sphere.

If you slice a sphere in half you will get two ________________, as shown below.

FINDING VOLUME AND SURFACE AREA OF CYLINDERS

The base of a cylinder is a circle. The volume of a cylinder is calculated by
multiplying the area of the base by the height of the cylinder. The formula for
calculating the volume of a cylinder is:

\[ V = \pi r^2 h \]
EXAMPLE 7: Find the volume of a cylinder with a diameter of 3 inches and a height of 5 inches. State your answer in terms of \( \pi \) and round your answer to the nearest tenth.

GUIDED PRACTICE

Use the given dimensions to find the volume of each cylinder.

a) Radius = 5 cm, height = 7.5 cm, round your answer to the nearest tenth.

b) Diameter = 7 inches, height = 5 inches, state your answer in terms of \( \pi \).
SURFACE AREA OF A CYLINDER

A = area of 2 bases + area of curved surface
A = $2\pi r^2 + 2\pi rh$

EXAMPLE 8: A cylinder has a height of 15 centimeters and a radius of 4 centimeters. What is the surface area of the cylinder? State your answer in terms of $\pi$ and round to the nearest tenth.

GUIDED PRACTICE

a. A cylinder has a radius of 4 inches and a height of 7 inches. Find the surface area of the cylinder.

b. A cylinder has a radius of 6 feet and a height of 10 feet. Find the surface area of the cylinder. State your answer in terms of $\pi$. 
FINDING THE VOLUME OF PYRAMIDS AND CONES

The diagrams show a square pyramid, a rectangular pyramid, and a triangular pyramid.

Before you find the volume of a pyramid, you have to find the ____________. Then you can use the formula below. In the formula, B is the area of the base, and h is the height.

**VOLUME OF A PYRAMID**

\[ V = \frac{1}{3} B h \]

The volume of a pyramid is one-third the volume of a prism with the same height and base area as the pyramid. Similarly, the volume of a cone is one-third the volume of a cylinder with the same height and base area as the cone.

**VOLUME OF A CONE**

\[ V = \frac{1}{3} Bh \]
EXAMPLE 9: What is the volume of the rectangular pyramid?

GUIDED PRACTICE

a) The base of a pyramid is a right triangle. The triangle has a base of 5 centimeters and a height of 3 centimeters. The pyramid has a height of 6 centimeters. What is the volume of the pyramid?

**STEP 1:** Find $B$, the area of the base, using the area formula of a triangle:

$$B = \frac{1}{2}bh$$

**STEP 2:** Substitute the area of the base from part 1 and the height of the pyramid into the formula:

$$V = \frac{1}{3}Bh$$
b) A square pyramid has a height of 12 meters. If a side of the base measures 5.5 meters, what is the volume of the pyramid?

**STEP 1:** Find $B$, the area of the base, using the area formula of a square:

$$B = s^2$$

**STEP 2:** Substitute the area of the base from part 1 and the height of the pyramid into the formula:

$$V = \frac{1}{3} Bh$$

**EXAMPLE 10:** Find the exact volume of the cone. State your answer in terms of $\pi$. Also find an approximation of the volume of the cone, rounded to the nearest tenth.

** Exact Volume: ______________  Approximation: ____________  **
a) A party hat is in the shape of a cone. Find the exact volume of the party hat as well as an approximation of the volume of the cone. Round your answer to the nearest tenth.

The exact volume of the hat is ________________.

An approximation of the volume of the hat is ________________.

b) The diagram shows a cone-shaped container. Find the exact volume of the container as well as an approximation of the volume rounded to the nearest tenth.
EXAMPLE 1: Suzanne is serving yogurt in cone-shaped parfait glasses. The cone-shaped part of each glass has a height of 9 centimeter and a diameter of 7 centimeters. How many cubic centimeters of yogurt can Suzanne serve if she fills 3 parfait glasses? Round your answer to the nearest tenth.

GUIDED PRACTICE

Jean wants to sell lemonade in cone-shaped paper cups. Each cup has a diameter of 4 centimeters and a height of 8 centimeters. She wants to make enough lemonade for 50 cups. How much lemonade does Jean need to make? Round your answer to the nearest tenth.
FINDING THE SURFACE AREA OF PYRAMIDS AND CONES

The length from O to A on a ______________ circular cone is called the ______________ of the cone. The slant height is the distance between the vertex and a point on the _____________________________ of the base. When you cut the cone along OA and flatten it, you will get a portion of a circle with center O and radius \( \ell \) units.

To find the surface area, you find the ___________ of the area of its base and the area of its curved surface. The curved surface is called the lateral surface.

The area of the base of a cone is ________, where \( r \) is the radius of the cone.

The area of the lateral surface is ________, where \( \ell \) is the slant height of the cone. Lateral means side, so the lateral surface area of a prism, pyramid, cylinder, and cone does not include the area of the base (or bases).
SURFACE AREA OF CONE

\[ A = \pi r^2 + \pi rl \]

EXAMPLE 11: The diagram shows a cone with a radius of 5 centimeters and a slant height of 15 centimeters.

PART A: Find the exact lateral surface area (curved surface) of the cone.

PART B: Find the total surface area of the cone. Find the exact surface and round your answer to the nearest tenth.
GUIDED PRACTICE

A solid cone has a radius of 7 inches and a slant height of 14 inches.

**PART A:** What is the exact area of the cone’s curved surface?

**PART B:** What is the total surface area of the cone? Find the exact value and round your answer to the nearest tenth.
EXAMPLE 12: A cone-shaped roof has a radius of 14 feet and a slant height of 17 feet. The roof is covered completely with glass. The cost of glass is $40 per square foot. What is the cost of covering the roof with glass?

GUIDED PRACTICE

Jessica makes a cone-shaped paper filter to line a cone-shaped funnel. The funnel has a radius of 5 inches and a slant height of 10 inches. Suppose Jessica wants to make cone-shaped filters for 25 such funnels. About how many square inches of filter paper will she need?
FINDING THE VOLUME OF SPHERES

The volume of a sphere with a radius of r units is given by the formula below. The diameter of a sphere is a segment that passes through the center of the sphere and has its endpoints on the surface of the sphere. The diameter is twice the length of the radius.

**VOLUME OF A SPHERE**

\[ V = \frac{4}{3} \pi r^3 \]

**EXAMPLE 13:** An iron ball has a diameter of 6 centimeters. Find the exact volume of the iron ball and round your answer to the nearest cubic centimeter.

The exact volume of the sphere is _______ and the approximation is _____.

The exact volume of the sphere is _______ and the approximation is _____.

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GUIDED PRACTICE

The diameter of a sphere is 8.8 meters. What is the exact volume of the sphere? What is the approximate value rounded to the tenths place.

The exact volume is ___________ and the approximation is ___________.

FINDING THE SURFACE AREA OF SPHERES

SURFACE AREA OF A SPHERE

\[ S = 4\pi r^2 \]
EXAMPLE 14: A sphere has a radius of 3 centimeters. What is the surface area of the sphere? Find the exact surface area and round your answer to the nearest tenth.

GUIDED PRACTICE

What is the surface area of a sphere with a radius of 6 centimeters? Find the exact surface area and round your answer to the nearest tenth.